Using Multidimensional Solvers for Optimal Data Partitioning on Dedicated Heterogeneous HPC Platforms

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Outline

- Data-intensive parallel computational routines: computational workload is divisible and proportional to data size
  number of processors: p
  data partition: $n = d_1 + d_2 + \cdots + d_p$
- Dedicated heterogeneous HPC platforms: load is balanced when the execution times are equal:
  $t_1 = t_2 = \cdots = t_p$

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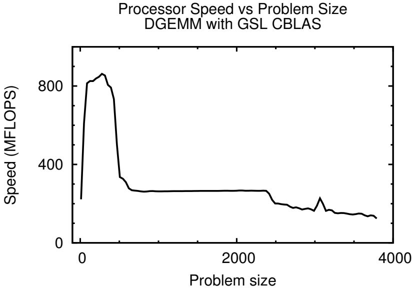
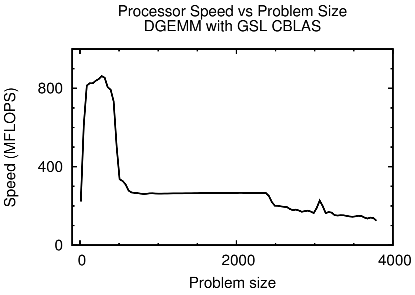
  $t_1 = t_2 = \cdots = t_p$
- Processor speed: $s_i = \dfrac{d_i}{t_i}$
- Data partitioning problem:

$$\begin{cases} \dfrac{d_1}{s_1} = \dfrac{d_2}{s_2} = \cdots = \dfrac{d_p}{s_p} \\ d_1 + d_2 + \cdots + d_p = n \end{cases} \tag{1}$$

- Traditionally, processor performance is defined by a constant number: $s = const$
- In reality, speed is a function of problem size: $s = s(x)$



- Traditionally, processor performance is defined by a constant number: $s = const$
- In reality, speed is a function of problem size: $s = s(x)$
- Partitioning algorithms based on constant performance models are only applicable for limited problem sizes
- How to solve (1) with speed functions?



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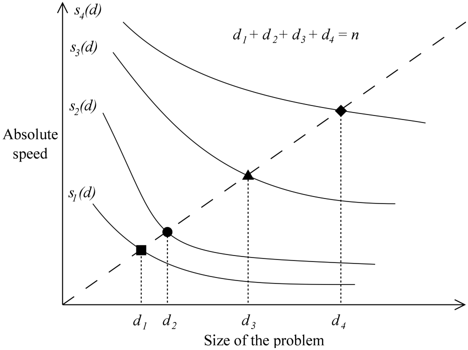
Geometrical solution: points $(d_i, s_i(d_i))$ on a line passing through the origin $\Rightarrow \dfrac{d_i}{s_i(d_i)} = const$ and $\sum d_i = n$



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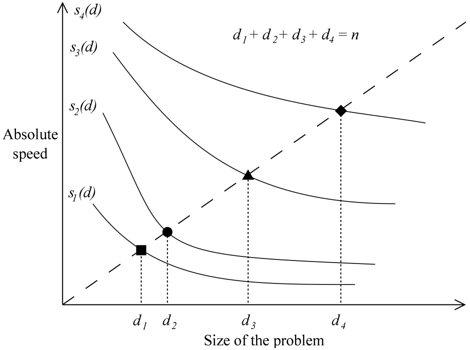
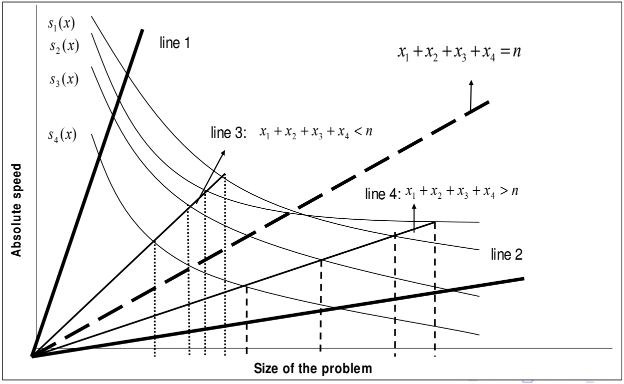


(2) Assumption: any straight line passing through the origin intersects speed functions only once

- The space of solutions: all lines passing through the origin
- Initial bounds for some $n^U < n$ and $n^L > n$:
  $x_1^U, \dots, x_p^U$: $\sum x_i^U = n^U$, and $x_1^L, \dots, x_p^L$: $\sum x_i^L = n^L$
- The region between two lines is iteratively bisected and the bounds are updated



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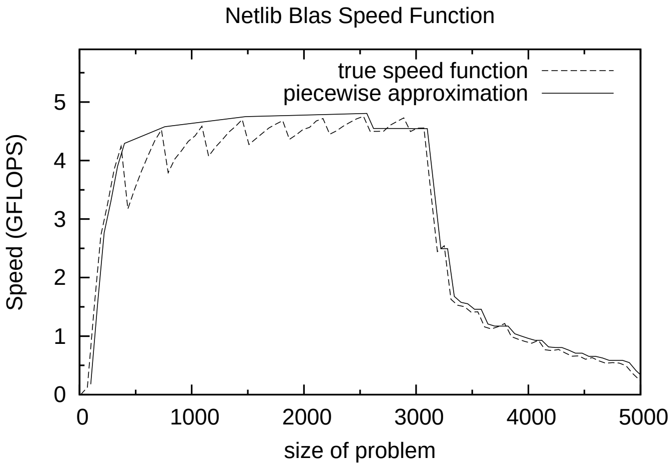
Restrictions on the shape of a speed function $s(x)$ to satisfy the assumption (2):

- Increasing and convex on $[0, X]$
- Decreasing on $[X, \infty]$

Fixes to the piecewise linear interpolation $\bar{s}(x)$ after adding a new data point (d^j, s^j):

- if $d^j \in [0, X]$, ensure that $s^{j-1} \leq s^j \leq s^{j+1}$ and $\dfrac{s^{j-1} - s^{j-2}}{d^{j-1} - d^{j-2}} \geq \dfrac{s^j - s^{j-1}}{d^j - d^{j-1}} \geq \dfrac{s^{j+1} - s^j}{d^{j+1} - d^j}$
- if $d^j \in [X, \infty]$, ensure that $s^{j-1} \geq s^j \geq s^{j+1}$

Result: inaccurate approximation of speed function



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- Data partitioning problem (1) can be formulated as **multidimensional root finding** for the system of nonlinear equations $f(\mathbf{x}) = 0$, where

$$f(\mathbf{x}) = \begin{cases} n - \sum_{i=1}^{p} x_i \\ \frac{x_i}{s_i(x_i)} - \frac{x_1}{s_1(x_1)} & 2 \leq i \leq p \end{cases} \tag{3}$$

  $\mathbf{x} = (x_1, ..., x_p) \in \mathbb{R}^p$ represents data partition
- Optimal data partition is obtained after rounding of the root $\mathbf{x}^* = (x_1^*, \ldots, x_p^*)$ and distribution of the remainders

- Problem (3) can be solved by the Newton-Raphson method:

$$\mathbf{x}^{k+1} = \mathbf{x}^k - J(\mathbf{x}^k) f(\mathbf{x}^k) \tag{4}$$

- Initial guess: the equal data distribution

$$\mathbf{x}^0 = (n/p, \dots, n/p) \tag{5}$$

- Jacobian $J(\mathbf{x})$: (6)

$$J(\mathbf{x}) = \begin{pmatrix} -1 & -1 & \dots & -1 \\ -\frac{s_1(x_1) - x_1 s_1'(x_1)}{s_1^2(x_1)} & \frac{s_2(x_2) - x_2 s_2'(x_2)}{s_2^2(x_2)} & 0 & 0 \\ \dots & 0 & \dots & 0 \\ -\frac{s_1(x_1) - x_1 s_1'(x_1)}{s_1^2(x_1)} & 0 & 0 & \frac{s_p(x_p) - x_p s_p'(x_p)}{s_p^2(x_p)} \end{pmatrix}$$

To solve (4)-(6), we use the HYBRJ algorithm, a modified version of Powell's Hybrid method, implemented in the MINPACK library:

- retains the fast convergence of the Newton method
- reduces the residual when the Newton method is unreliable
- requires differentiable speed functions

Layout

Approximations of speed function

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- Splines of higher orders: differentiable but may yield significant oscillations

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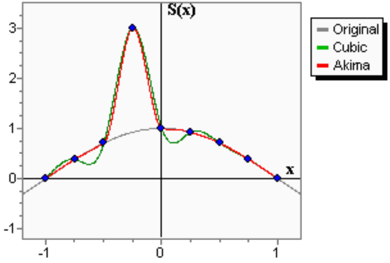
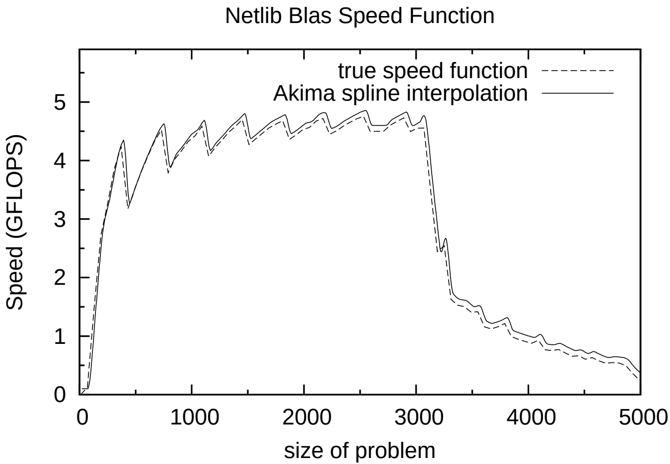
- Splines of higher orders: differentiable but may yield significant oscillations
- **Akima spline interpolation**: non-linear but stable to outliers



Akima spline interpolation

- Built from piecewise third order polynomials
- Computationally efficient:
  - no need to solve large equation systems
  - a small number of the neighbour points is taken into account
- Interpolation error in the inner area $O(h^2)$

Result: accurate approximation of speed function



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Parallel Computational Iterative Routine

$$\mathbf{x}^{k+1} = f(\mathbf{x}^k) \qquad \mathbf{x}^k \in \mathbb{R}^n \qquad f : \mathbb{R}^n \to \mathbb{R}^n$$

- Data is partitioned over all processors: $n = d_1^k + d_2^k + \cdots + d_p^k$
- Some independent calculations are carried out in parallel
- Some data synchronisation takes place

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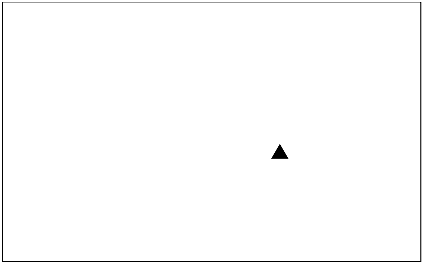
Model-based Dynamic Load Balancing

- At each iteration, execution times measured and sent to root
- Approximations of speed functions $\bar{s}_i(x)$ updated by adding the point $(d_i^k, d_i^k/t_i^k)$
- If relative difference between times $> \epsilon$, data partitioning algorithm calculates new data partition $\mathbf{d}^{k+1}$
- $\mathbf{d}^{k+1}$ broadcasted to all processors; data redistributed

Layout

First iteration Point $(n/p, s_i^0)$ with speed $s_i^0 = \frac{n/p}{t_i(n/p)}$

First function approximation $\bar{s}_i(x) \equiv s_i^0$



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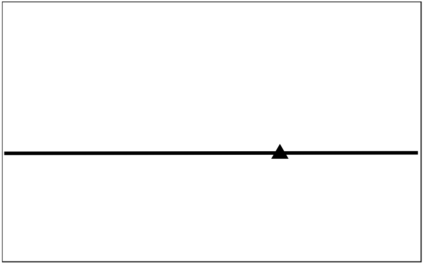

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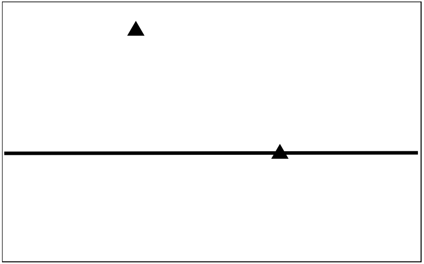
Subsequent iterations Point (d_i^k, s_i^k) with speed $s_i^k = \dfrac{d_i^k}{t_i(d_i^k)}$

Approximation $\bar{s}_i(x)$ updated by adding the point



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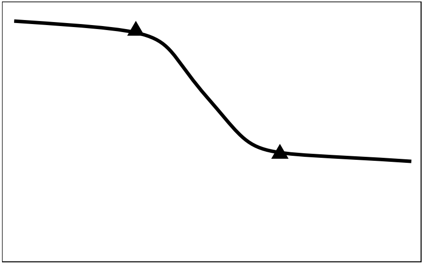

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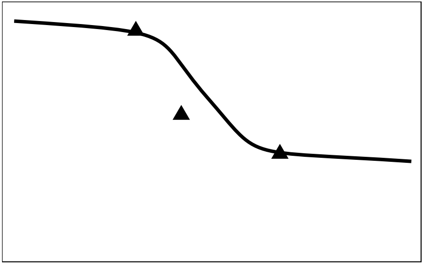

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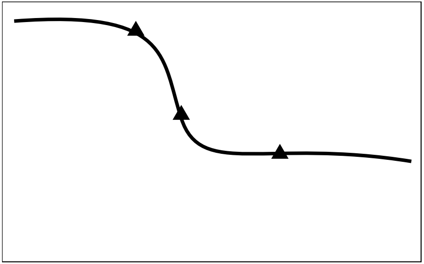

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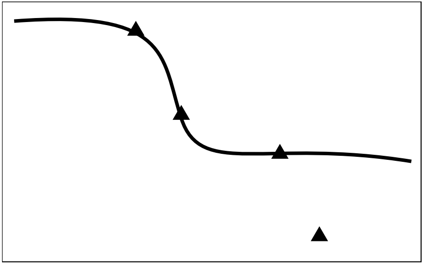

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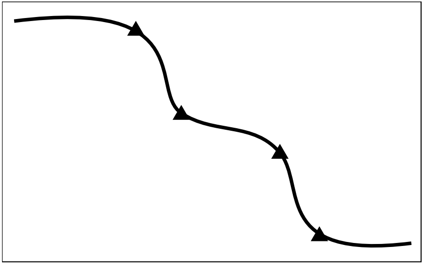

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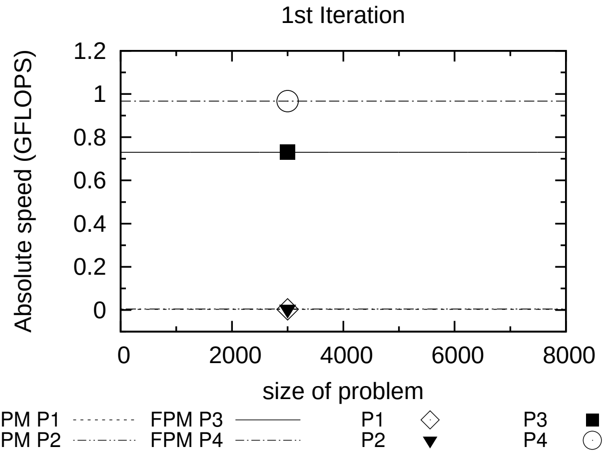
Experimental Results: Jacobi Method



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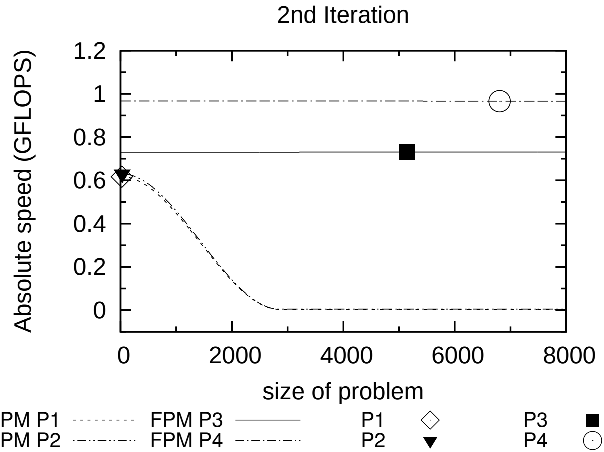

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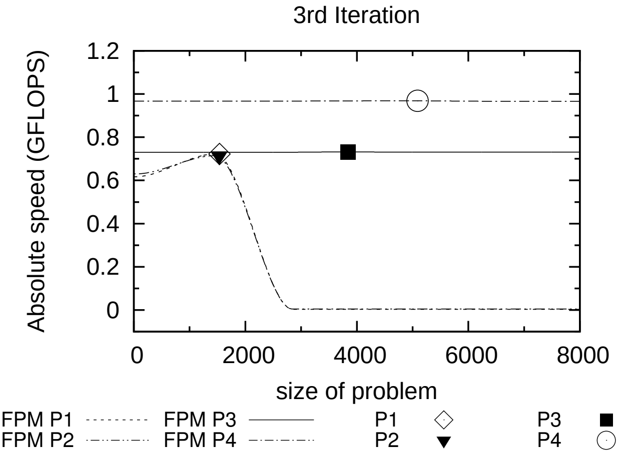

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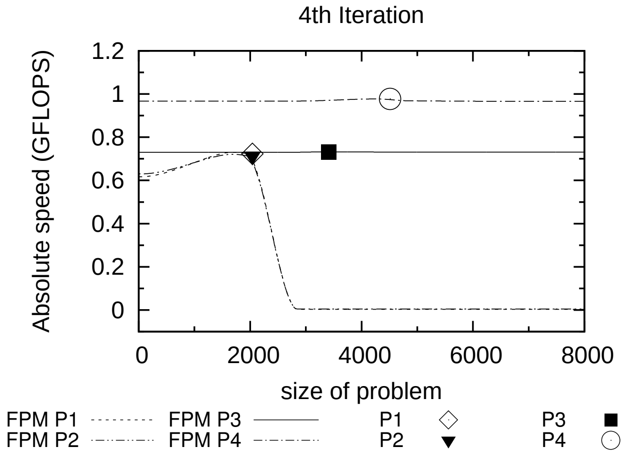

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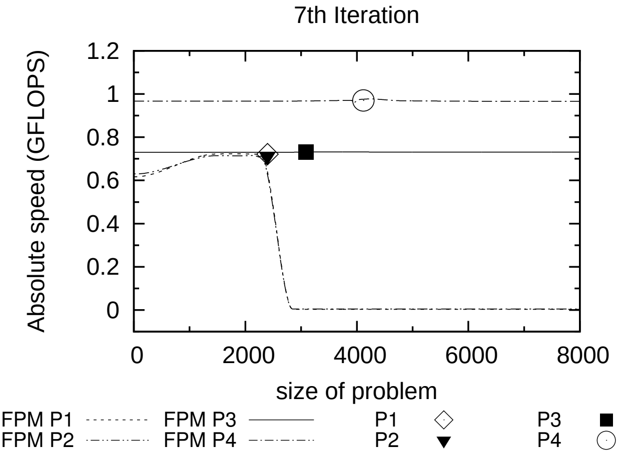

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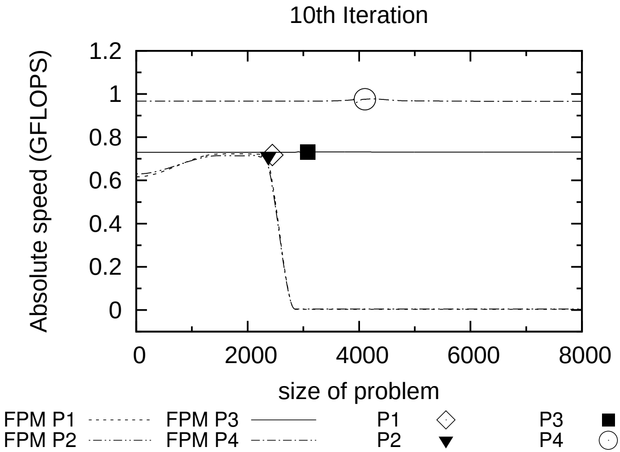

Conclusions

- Traditional data partitioning algorithms only work for problems which fit into the main memory of all processors.
- The proposed algorithm, based on accurate functional performance models, can balance for all problem sizes.
- No prior information about the heterogeneity and memory hierarchy of the platform needed as inputs into the algorithm.
- Can be deployed self adaptively on any dedicated platform.

Project web page: `http://hcl.ucd.ie/project/fupermod`

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