Column-Based Matrix Partitioning for Parallel Matrix Multiplication on Heterogeneous Processors Based on Functional Performance Models

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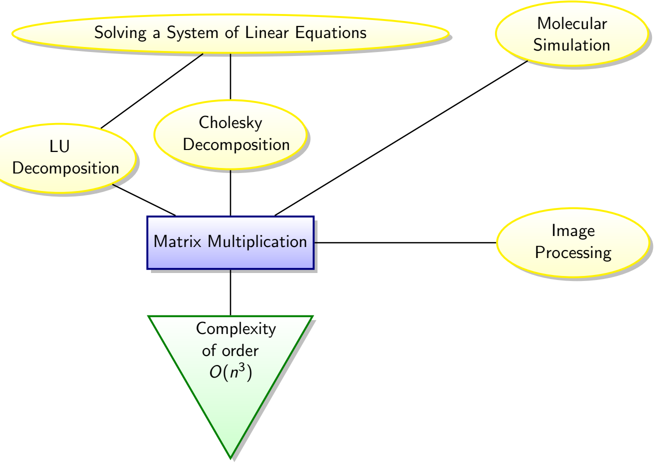
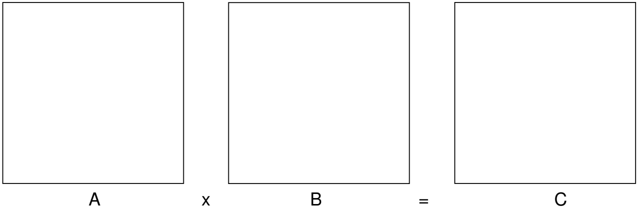
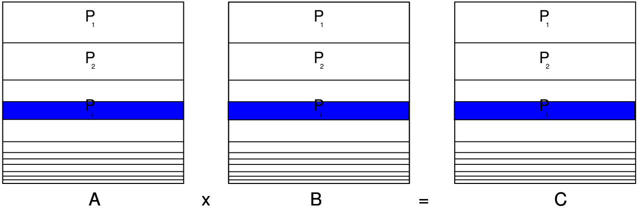
HeteroPar'2011

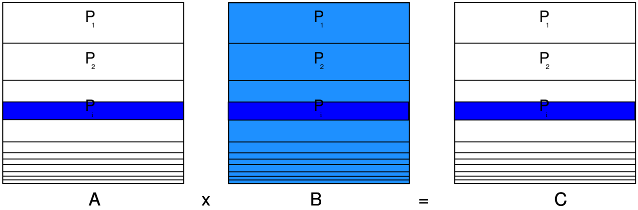
Outline

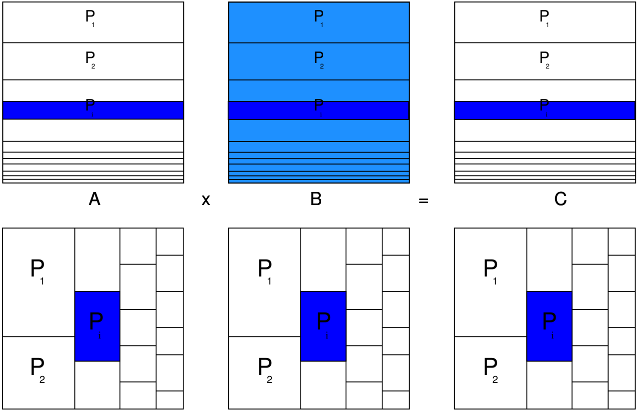
Why optimize Matrix Multiplication?

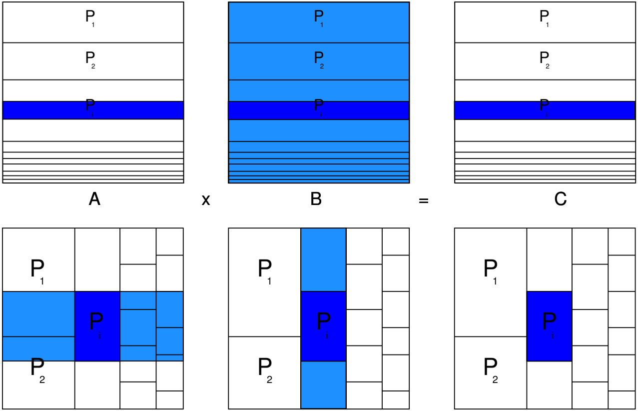



A
x
B
=
C


P
1
P
2
P
i
A
x
B
=
C

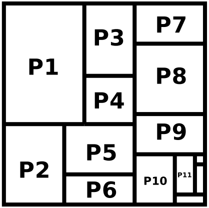

P1
P2
Pi
A
x
B
=
C


P1
P2
Pi
A
x
B
=
C
P1
P2
Pi

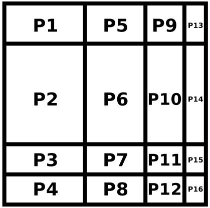

P 1
P 2
P i
A
x
B
=
C

Optimising Parallel Matrix Multiplication on a Heterogeneous Platform

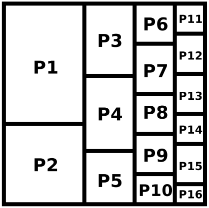
- Partition in proportion to processor speed.
- Minimise volume of communication.
- Partition in proportion to interconnect speed.



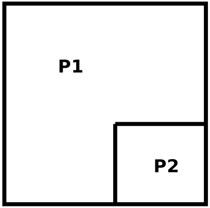
General (NP complete)



Cartesian

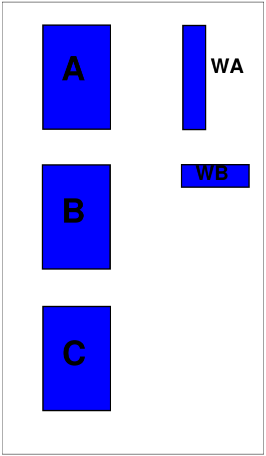


Column-Based



Square-Corner

allocate and initialise matrices A, B, C;
allocate workspace WA, WB;
for $k = 0 \rightarrow N - 1$ **do**
  if (is pivot row) **then**
    point WB to local pivot row of B;
    Broadcast WB to all in column;
  else
    Receive WB;
  end if
  if (is pivot column) **then**
    point WA to local pivot column of A;
    Send WA horizontally;
  else
    receive WA;
  end if
  DGEMM($\ldots$, WA, WB, C, $\ldots$);
end for



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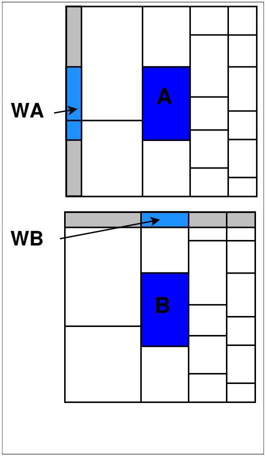

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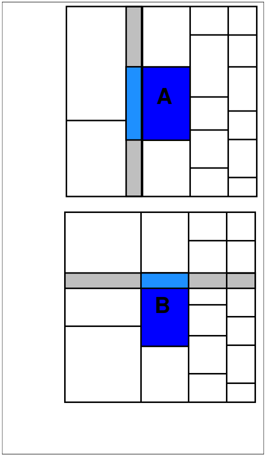


Benchmarking on each processor must be independent of other processors: serial code.

allocate and initialise matrices A, B, C;
allocate workspace WA, WB;

start timer;
MPI_Send(A, ..., MPI_COMM_SELF);
MPI_Recv(WA, ..., MPI_COMM_SELF);
memcpy(WB, B, ...);
DGEMM(..., WA, WB, C, ...);
stop timer;

free memory;

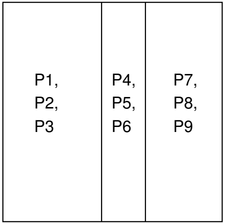
Matrix Partitioning Algorithms

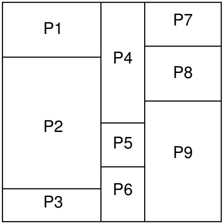
- Column-Based Partitioning
  (Kalinov & Lastovetsky 1999) (**KL**)
- Minimising Total Communication Volume
  (Beaumont, Boudet, Rastello, Robert, 2001) (**BR**)
- 1D Functional Performance Model-based Partitioning
  (Lastovetsky, Reddy, 2007) (**FPM1D**)
- 2D Functional Performance Model-based Partitioning
  (Lastovetsky, Reddy, 2010) (**FPM-KL**)

- New Two-Dimensional Matrix Partitioning Algorithm
  (**FPM-BR**)

Column-Based Partitioning (**KL**)

- Processors are arranged into columns.
- The width of each column is in proportion to the sum of the speeds of the processors in that column.
- Within each column the heights are calculated in proportion to speed.





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- Within each column the heights are calculated in proportion to speed.

- However, communication cost is not taken into account.
- Uses inaccurate, single-value performance model of processor speed.

Minimising Total Communication Volume (**BR**)

- Column-based algorithm.
- Computes:
  - Optimum number of columns
  - Optimum number of processors in each column
- Such that:
  - Workload is distributed in proportion to speed,
  - Total volume of communication is minimised.

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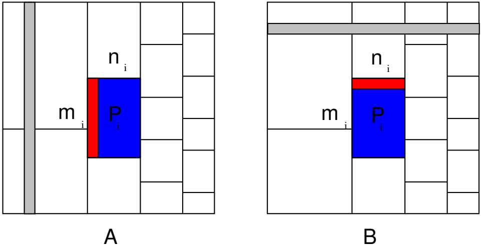
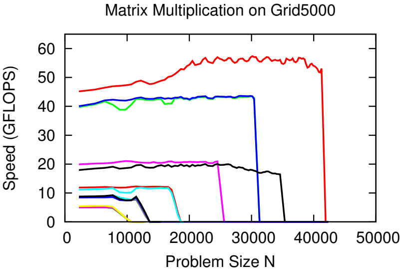


Total volume of communication $= \sum_i^p (m_i + n_i)$
"the sum of the half perimeters"
minimised when $m_i \approx n_i$

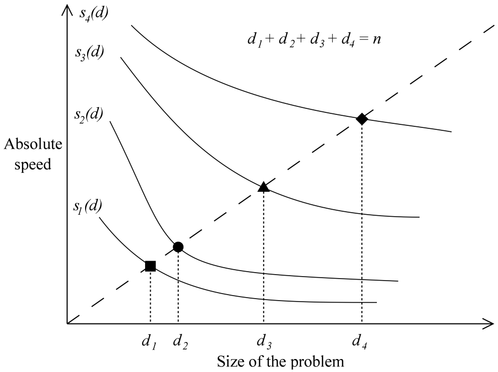
Realistic Performance Models

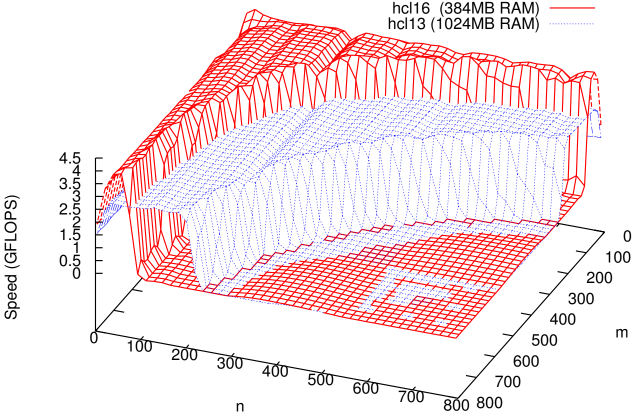
- Traditionally, processor performance is defined by a constant number.
- In reality, speed is a function of problem size.
- Algorithms based on constant performance models are only applicable for limited problem sizes.



1D Functional Performance Model-based Partitioning (**FPM1D**)

- Problem is solved geometrically by noting that the points $(d_i, s_i(d_i))$ lie on a line passing through the origin when $\frac{d_i}{s_i(d_i)} = constant$.



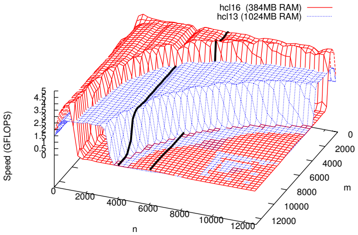

hcl16 (384MB RAM)
hcl13 (1024MB RAM)
Speed (GFLOPS)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
n
0
100
200
300
400
500
600
700
800
m
0
100
200
300
400
500
600
700
800

2D Functional Performance Model-based Partitioning (**FPM-KL**)

- Column-based partitioning with 2D performance models.
- Processors are arranged in a grid $p \times q$
- Column widths are initially distributed $n_j = N/q \ \forall j$.

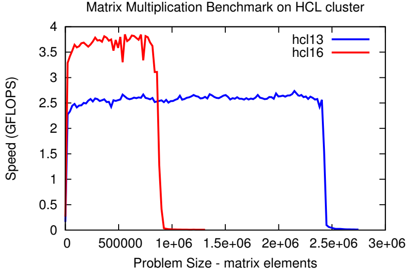
Iterating:

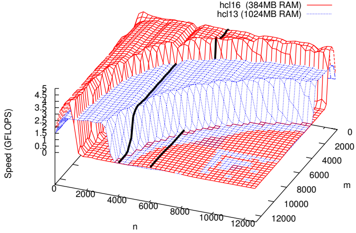
1. 1D models are sliced from 2D at column widths.
2. Optimum partitioning within each column is solved with **FPM1D** algorithm.
3. If disbalance $< \epsilon$ then finished, else continue.
4. Single value speeds from this partitioning used to calculate new column widths.



2D Functional Performance Model-based Partitioning (**FPM-KL**)

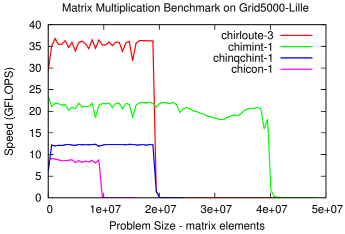
- Does not take communication cost into account.
- Processor grid is fixed.
- Relies on single speed values to calculate new column widths.
- Building full 2D models is expensive

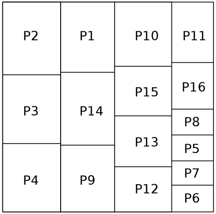




New Two-Dimensional Matrix Partitioning Algorithm (**FPM-BR**)

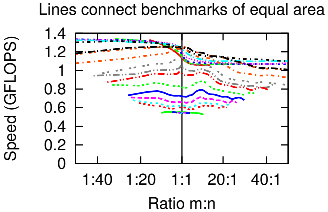
- Height m_i and width n_i combined into one parameter, area $d_i = m_i \times n_i$.
- Square areas are benchmarked $m = n = \sqrt{d}$.
- Partition with **FPM1D** algorithm, find area rectangles.
- BR algorithm computes ordering and shape of these rectangles.

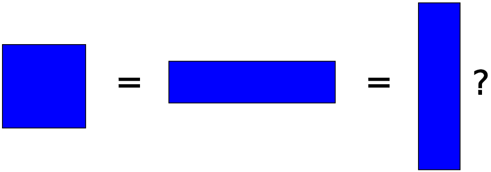




Assumption square area performance is the same as performance of any rectangle of the same area, $s(x,x) = s(x/c, c.x)$.

- not always true.





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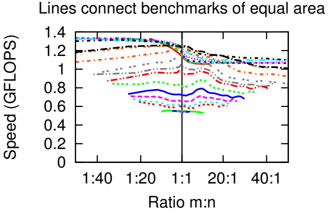
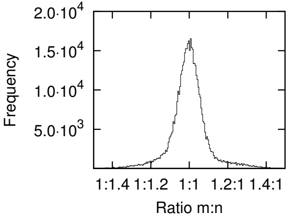
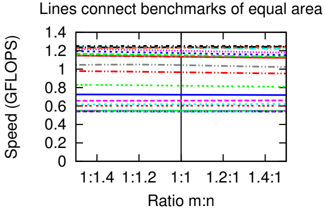
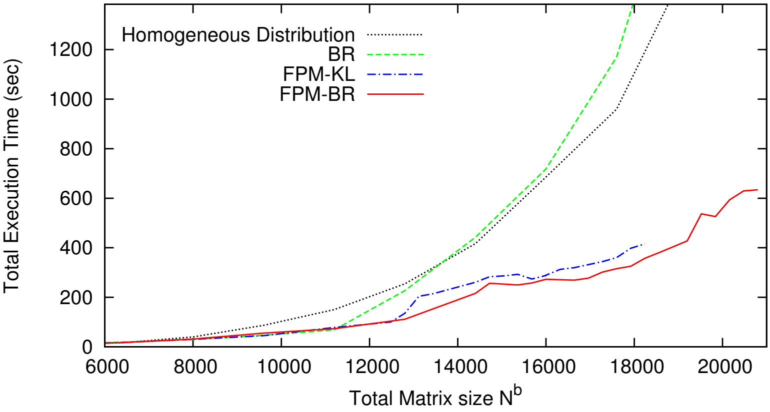


However, goal of **BR** algorithm is to make rectangles square.

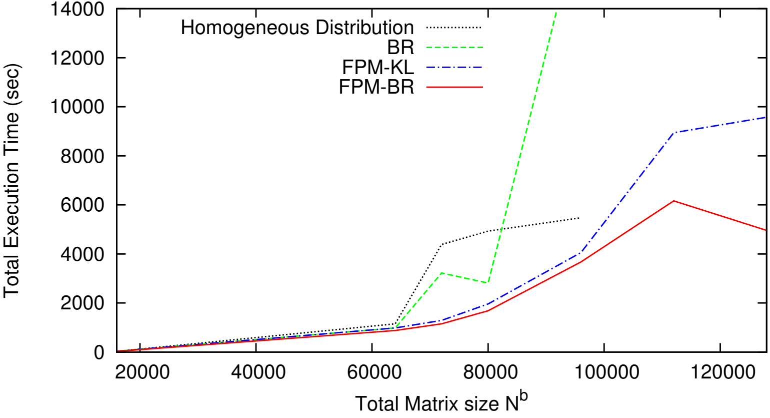




Experimental Results



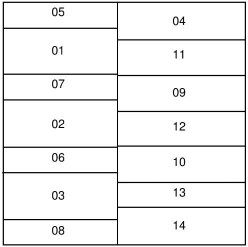
16 heterogeneous nodes, local HCL cluster.



64 nodes from Grid5000 Lille site (4 types of nodes).

Matrix partitioning for 14 nodes

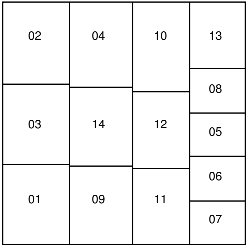
FPM-KL



TVC: 9

Time: 192.2sec

FPM-BR



TVC: 7.457

Time: 166.0sec

Conclusions

- New **FPM-BR** algorithm can outperform existing algorithms.
- Allows use of simpler 1D models.
- Total volume of communication is minimised.

Questions?